Multi-dimensional Numerical Fourier Analysis of Disturbing Function

SERGEI KLIONER
Lohrmann Observatory, Dresden Technical University,
Mommsenstraße 13, 01062 Dresden, Germany
e-mail: klioner@rcs.urz.tu-dresden.de

It is well known that Fourier expansions of planetary disturbing function can be computed numerically with the help of numerical Fourier analysis (see, e.g., Brouwer, Clemence, 1961). What is making this approach advantageous nowadays is fast computers and Fast Fourier Transformation (FFT) algorithm. The FFT algorithm has been introduced into modern practice by Cooley and Tukey in 1965. Retrospectively it has become known that the FFT algorithm was invented independently by a dozen of individuals starting from Gauss in 1805 (Heideman, Hohnson, Burrus, 1985). It was exciting also to learn that Gauss has invented the algorithm to compute Fourier expansions of planetary disturbing function. Most general 5-dimensional Fourier expansion of planetary disturbing function in standard notations reads:

$$R = \frac{G\,\mathcal{M}'}{a'} \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=0}^{\infty} B_{ijklm}(\alpha, e, e', i, i') \times \times \cos(i\,M + j\,M' + k\,\omega + l\,\omega' + m(\Omega - \Omega')), \quad (1)$$

$\alpha = a/a'$ being the ratio of the two semi-major axes. Numerical Fourier transformation of R for any admissible numerical values of α, e, e', i, i' allows one to compute coefficients B_{ijklm}. Numerical computation of 5-dimensional series (1) has become feasible on a typical server-class computer quite recently, the main problem being the large amount of required RAM. It is clear that due to intrinsic aliasing errors of the FFT for any given amount of memory there exists a region in the 5-dimensional parameter space, for which B_{ijklm} can be computed with a given accuracy. For practical calculations we used SGI Origin2000 with 48 R10000 processors running at 195MHz. On that computer we could use up to 8Gb RAM for a single calculation. To give an example, in order to compute series (1) for the perturbations of Jupiter on Veritas ($\alpha \approx 0.609$, $e \approx 0.1009$, $e' \approx 0.0485$, $i \approx 9°16'$, $i' \approx 1°18'$) with an accuracy of 10^{-14} the FFT size should be taken as $56 \times 52 \times 48 \times 24 \times 208$ which is required about 5.4 Gb RAM and 30 minutes of computing time in single processor mode. In this example, the number of coefficients B_{ijklm}, absolute value of which is greater than 10^{-14}, is 1 083 126. Our experience shows that that calculation of the most general Fourier expansions of the planetary disturbing function by a direct 5-dimensional FFT is quite feasible and can become a useful tool of celestial mechanics in the nearest future. More details can be found in Klioner (1999).

REFERENCES

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