INVESTIGATION OF AERATED SIPHON

Detlef Aigner, Hans-B. Horlacher

Institute for Hydraulic Engineering and Applied Hydromechanics,
Dresden University of Technology, 01062 Dresden, Germany,
phone: +49 351 46334397; fax: +49 351 46337120; e-mail: iwd@tu-dresden.de

ABSTRACT

Siphon is the name given to pressure conduits whose highest point (the crest) is situated above the water level of the reservoir (Figure 1). When the air inside them is evacuated either automatically or artificially, the siphon begins to work. Due to the low pressure at the crest, by regulating the aeration via air inlets the discharge can be regulated or stopped altogether. Siphons are typically employed as flood discharge. Non-aerated siphons are somewhat controversial due to the fact that, in a very short time, their maximum discharge is attained which then causes the reservoir water level to suddenly rise. The main objective of an aerated siphon is to have a regulating effect on the siphon discharge performance. In this paper we propose a simple method for the calculation and management of such a regulatory process.

Keywords: spillway, flood, siphon, air induction, water/air mixture

1 THEORY

A siphon for use in flood relief at reservoirs is usually constructed with a rectangular cross section area A, with a breadth b and a height a. The inlet is rounded off so as to cause little energy loss. Contraction is to be found in the vicinity of the siphon exit which prevent the uptake of air and, in so doing, facilitate priming. Staggered siphons are positioned at different heights so that they – in the case of an ever increasing water level – can begin to function at differing times.

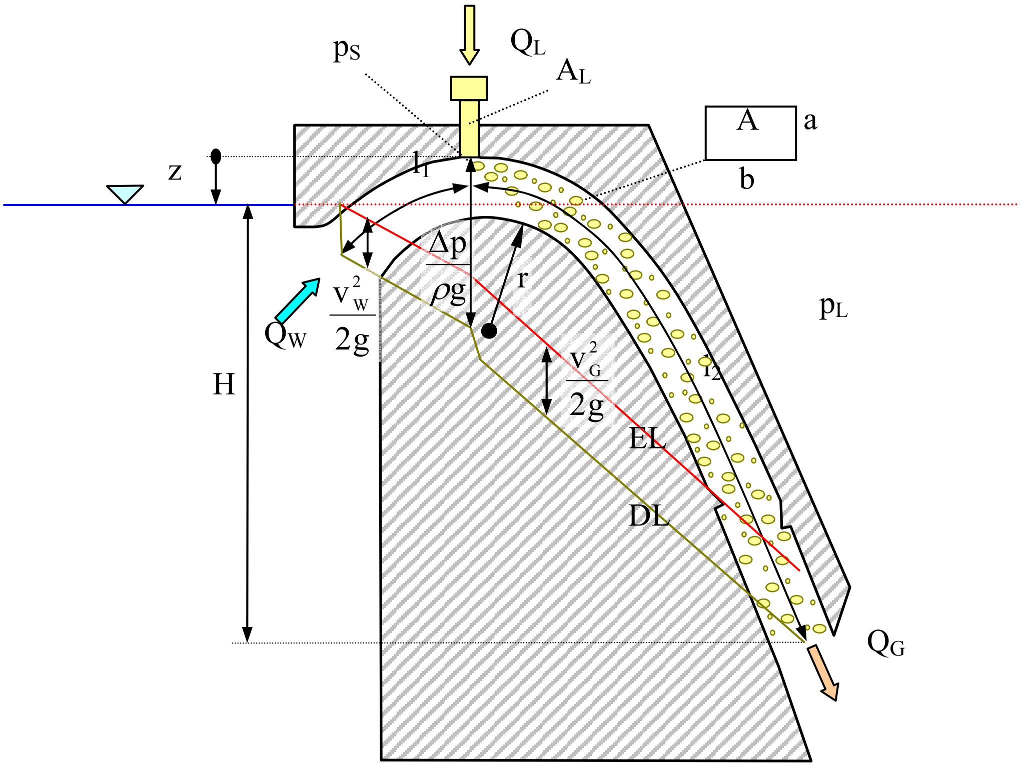

Figure 1: The principles of an aerated siphon

Situated in the crest of an aerated siphon there is a valve for regulating the air intake. When experimenting with physical models, the air inflow is often wrongly interpreted due to the fact that insufficient attention is paid to the relationship between the siphon's air requirements and the actual air supply available.

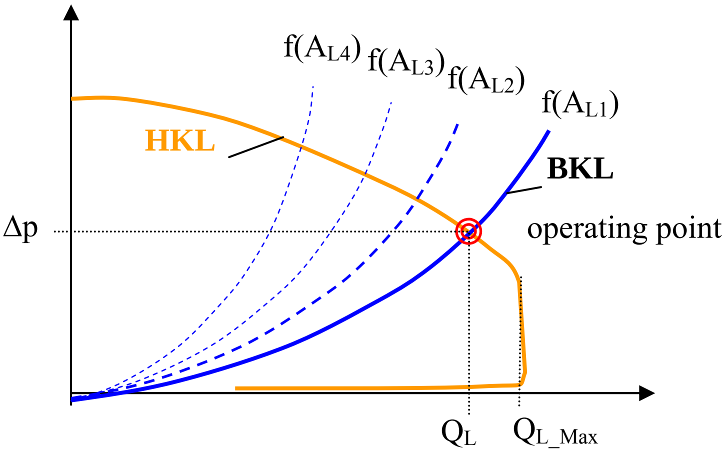

Figure 2: Siphon (HKL) and air inlet (BKL) characteristics. Operating point as function of the air inlet valve opening (A_L)

Figure 2 shows the typical characteristics for the calculation of the air flow in a siphon. The siphon characteristic demonstrates the progressive reduction of the low pressure Δp in the siphon crest with increasing air intake. The aeration characteristics define the degree of air intake in dependence of low pressure and the effective degree of openness $\mu \cdot A_L$ of the aeration valve including the energy losses incurred in the aeration canal. The intersection of the lines is the operating point for a specific degree of opening of air valve.

1.1 Definitions and Assumptions

Our calculations were the basis for the following definitions and assumptions:

Q_W	Water discharge[m^3/s],
Q_L	Air flow [m^3/s],
$\dot{m}_L = \rho_L \cdot Q_L$	Mass volume flow of the air [kg/s],
$Q_G = Q_L + Q_W$	Air and water discharge [m^3/s],
$ß = \frac{Q_L}{Q_W}$	Degree of aeration [-],
$v_W = \frac{Q_W}{A}$	Average velocity [m/s] in the siphon front of air valve,
v_{WS}	Velocity in the crest of the siphon bend [m/s],
$v_G = \frac{Q_G}{A}$	average, slipless velocity [m/s] of the mixture,

$\frac{v_G}{v_W} = ß + 1$ relativ velocity [-] of the mixture,

$A = b \cdot a$ Cross section of the siphon [m^2], rectangular cross section, crest value,

z Distance between the siphon crest and the water level [m],

H Energy head of the siphon [m],

r Siphon radius [m],

K Influence of the bend [-],

A_L Cross section of the aeration [m^2],

μ Inflow coefficient of the air flow including energy losses [-],

p_S Absolute atmospheric pressure in the siphon crest [Pa],

p_L Air pressure, Outside pressure [Pa],

L_1 Length of the siphon up to the aeration slots [m],

L_2 Length of the siphon after the aeration slots [m],

ρ_G Density of the mixture [kg/m^3],

ρ_L Air density [kg/m^3],

ρ_W Water density [kg/m^3],

R Gas constants of air [287,2 $m^2/s^2/K$],

T Temperature in Kelvin [K],

$d_{hy} = 4 \cdot r_{hy} = 4 \cdot \frac{A}{l_U} = 2 \cdot \frac{b \cdot a}{b + a}$ hydraulic diameter of A in [m],

$\lambda = f(\mathrm{Re}, \frac{k}{d_{hy}})$ Friction coefficient depending on the Reynolds-number and roughness [-],

$\sum \zeta$ Sum of the local energy losses [-],

$R_1 = \lambda_1 \cdot \frac{l_1}{d_{hy1}} + \sum \zeta_1$ Energy loss coefficients of the siphon up to the aeration valve [-],

$R_2 = \lambda_2 \cdot \frac{l_2}{d_{hy2}} + \sum \zeta_2$ Energy loss coefficients of the siphon after the aeration valve [-],

$\rho_L = \frac{p}{R \cdot T}$ Air density calculated via the gas equation [kg/m^3],

$\frac{\rho_G}{\rho_W} = \frac{1 + \frac{\rho_L}{\rho_W} \cdot ß}{ß + 1} \cong \frac{1}{ß + 1}$ Density ratio between the mixture and the water [-],

1.2 CALCULATION OF THE WATER FLOW UP TO THE AERATION

As is the case with normal pressure conduits, the hydraulic calculation of a siphon is done with an energy equation. If the cross section is not circular, the equivalent or hydraulic diameter d_{hy} is employed instead of the normal diameter. The energy equilibrium existing between the inlet and the aeration valve in the crest of the siphon results in the following equation (1).

$$p_L - p_S = \Delta p = \rho_W \cdot g \cdot (z + h_{V1}) + \rho_W \frac{v_{WS}^2}{2} = \rho_W \cdot g \cdot z + \rho_W \frac{v_W^2}{2} \cdot (R_1 + K) \qquad (1)$$

The pure water flow in the first section of the siphon defines the relationship between the distance z of the water level from the aeration opening (situated in the crest), the degree of low pressure at this point and, thirdly, the discharge. The velocity at the crest of the siphon bend v_{WS} is somewhat less than the average velocity v_W in the siphon. Their squared relationship defines the influence of the bend K. The friction coefficient R_1 considers not only the friction and local energy losses but also cross sectional differences within the siphon.

1.3 VELOCITY DISTRIBUTION DUE TO THE BEND

If the pressure and velocity distribution in the bend are considered within the crest cross section (Bollrich, 2000), the velocity head at the extreme boundary decreases in accordance with equation (2). The ratio between the reduced velocity head and the average velocity head is defined as the bend influence K. At the extreme edge of the crest – the point where the aeration takes place – K will always have a value lesser than 1.

$$K = \frac{v_{WS}^2}{v_W^2} = \left(\frac{1}{(r/a + 1) \cdot \ln(1 + a/r)}\right)^2 \qquad (2)$$

Turbulent pressure changes caused by changes in velocity can lead to the siphon closing down earlier than expected. In such a situation the pressure can fluctuate of the size of dynamic pressure (velocity head).

With assuming K = 0, this influence can be considered in calculations for determining the cut-off point of flow.

1.4 CALCULATION OF THE MASS FLOW RATE OF THE AIR

Due to the compressibility of the air, the air intake is calculated according to the equation (6) as mass flow rate (Will, Ströhl, 1990):

$$\dot{m}_L = \mu \cdot A_L \cdot \Psi \cdot \sqrt{2 \cdot \rho_L \cdot p_L} \qquad (6)$$

with $$\Psi = \sqrt{\frac{\kappa}{\kappa - 1} \cdot \left[\left(\frac{p_S}{p_L}\right)^{\frac{2}{\kappa}} - \left(\frac{p_S}{p_L}\right)^{\frac{\kappa+1}{\kappa}} \right]} = \sqrt{3{,}5 \cdot \left[\left(\frac{p_S}{p_L}\right)^{\frac{10}{7}} - \left(\frac{p_S}{p_L}\right)^{\frac{12}{7}} \right]} \qquad (6a)$$

As is the case with normal pressure conduits, the hydraulic calculation of a siphon is done with an energy equation. If the cross section is not circular, the equivalent or hydraulic

Ψ is the out-flow function for compressible gases, calculated with the adiabatic exponent $\kappa = 1{,}4$ for air. The inflow coefficient μ comprises the contraction and the energy losses of the in-coming air. The air density and the air volume flow are now different depending on whether they are at the siphon crest with very low pressure p_S or at the siphon out flow with normal atmospheric pressure. This has an influence on the ß-value and also affects the velocity of the mixture flow.

For a given pressure p_S in the siphon crest, the density of the sucked in air is calculated by the gas equation given in point 1.1.

The air volume flow Q_{LS} and thus also the degree of aeration $ß_S$ in the crest then becomes:

$$ß_S = \frac{Q_{LS}}{Q_W} = \frac{\dot{m}_L}{Q_W \cdot \rho_{LS}} = \frac{\dot{m}_L \cdot R \cdot T}{Q_W \cdot p_S} \tag{7a}$$

and for the exit at the end of the siphon at atmospheric pressure to:

$$ß = \frac{Q_L}{Q_W} = \frac{\dot{m}_L}{Q_W \cdot \rho_L} = \frac{\dot{m}_L \cdot R \cdot T}{Q_W \cdot p_L} \tag{7b}$$

For calculating the energy losses in the mixture along second section of the siphon, we calculated the averaged value $ß_m$ of the two air properties with:

$$ß_m = \frac{ß}{2} \cdot (1 + \frac{p_L}{p_S}) \tag{7c}$$

1.5 INFLOW COEFFICIENT OF THE AIR

The inflow coefficient μ comprises the energy losses and the contraction of the air flow. It is affected by the geometry of the inlet and also by the water flow itself within the crest of the siphon. The velocity of the water in the siphon causes a lateral deflection and contraction of the inflowing air. Aigner (2000) investigated how far the main stream influences the contraction of the water inflow. This influence would have the same effect for inflowing air, although the influencing role played by the density difference is unknown. The aeration valve used in the model was assigned an inflow coefficient of $\mu = 0,85$ by its manufacturers. A comparison between the measured in the model and calculated air inflow resulted in a inflow coefficient of $\mu = 0,4 + 10/3.z/H$.

1.6 ENERGY EQUATION FOR THE MIXTURE

As the air volume flow is pressure dependent, this leads to the air in the siphon crest occupying more space – due to the low pressure – than at the siphon exit where normal atmospheric pressure exists. On the assumption that there is a uniform distribution of the water and the air - in an attempt to simplify the procedure - the friction value is calculated on the basis of a mixture velocity which, itself, is derived from the averaged air density. It is only the velocity head at the siphon exit which is calculated with the mixture density for normal atmospheric pressure. For the friction in the turbulent mixture, the same coefficient of flow resistance was used as is valid for pure water flows. Comparable calculations for siphons without air can be used to determine the amount of the local energy losses. The additional energy losses due to mixing with the air (combination losses) and due to slipping (slipping losses) can be determined with the help of comparable calculations with measurements taken either from models or from nature.

The energy balance between the siphon crest and the exit provides:

$$\rho_G \cdot g \cdot (H+z) = \rho_W \cdot g \cdot z + \rho_W \frac{v_W^2}{2} \cdot R_1 + \rho_G \cdot \frac{v_G^2}{2} \cdot R_2 + \rho_{GE} \cdot \frac{v_{GE}^2}{2} \qquad (8)$$

The transforming of equation (8) with the definitions in point 1.1 provides an equation for calculating the velocity head with the velocity of the water flow. The discharge Q_W is determined from the continuity-equation.

$$\frac{v_W^2}{2g} = \frac{H - z \cdot ß_m}{(ß_m + 1) \cdot (R_1 + (ß_m + 1) \cdot R_2 + ß + 1)} \qquad (9)$$

It was quite a complicated matter to determine the loss of mixing between water and air as part of the friction losses R_2 in siphon.

We tried to quantify these losses with the help of other comparable calculations. For this energy loss coefficient we employed:

$$\zeta_V = 7 \cdot ß_m^2 \qquad (10)$$

Due to the fact that the equation system is dependent upon many influencing factors, this equation is only valid for the stated assumptions and stated examples.

2 EXAMPLES

The transfer and application of the results arising from experiments with modelled aerated siphons on to real life size siphons can become extremely complicated. Their scale up application to life-size siphons can only be successful if a comparison is made with measurements carried out on the original (see Bollrich/Aigner 2000). The different results achieved between the model and the life size siphon are a consequence of the limiting restrictions involved when scaling up or down to any great extent. These limits exist, for example, a) due to the respective atmospheric pressure at the reservoir which can't be realistically modelled, b) due to the unavoidable difference in the size of the air bubbles between real size and model siphons, c) due to the turbulence related differences with respect to the times when air uptake begins, d) due to the non-identical surface tension and the differing air compressibility and d) due to the non-identical energy losses between real size and model siphon.

During 1994 and 1995 at Dresden University's Institute for Hydraulic Engineering and Applied Hydromechanics modelled versions of the aerated siphons installed at the Oker dam were used in experiments for the Harz Waterworks (Horlacher, Dornack, Müller, 1995).

They were based on modelling experiments which were carried out by Prof. Press in Berlin during 1953 – 1955 in preparation for the construction of the reservoir. Unfortunately at that time, the results of the (somewhat simplified) modelling trials caused the aeration valve to have been built far too small. Even the results of the modelling trails in 1994 and 1995 provided values which were still unrealistically small. Based on comparisons with life size data it was then proposed to apply a up scaling factor of 300% for the aeration cross section.

For the siphon a constant energy loss coefficient of $\zeta_2 = 2{,}9$ was determined for the flow of pure water and this was allocated practically completely to the 2nd section of the siphon. It comprises the local energy losses occurring in the inlet, within the bend, due to local obstacles and also includes the effects of the cross sectional differences within the siphon channel (a=0,65…2,8m, b=1,32…1,5m). As for the friction value, an hydraulic roughness of k = 1mm was assumed for life size siphons and k = 0,1mm for models.

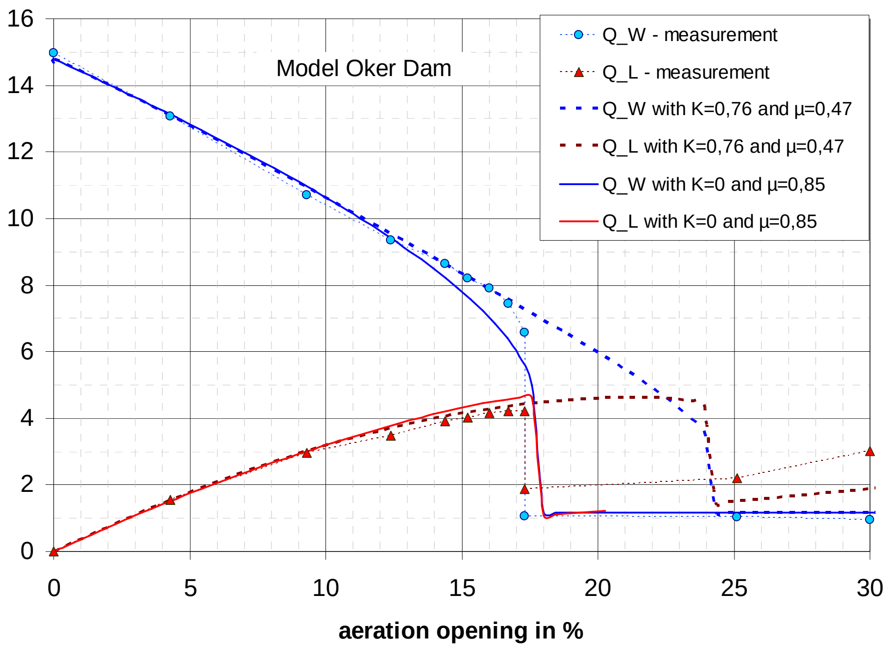

Figure 3: A comparison calculation with model values for a dynamic head of 417,03 m above SL for the Oker Dam

Assumptions and values of the example calculation for the model:

$Q_W = 0{,}03\ m^3$ $A = 0{,}015\ m^2$ $R_1 = 0{,}236$ $K = 0{,}76$
$A_L/A = 0{,}027$ $z = 0{,}047\ m$ $H = 2{,}25\ m$ $R_2 = 4{,}38$

The measured values of the hydraulic model were very satisfactorily reproduced for K = 0,76 and $\mu = 0{,}4 + 10/3 \cdot z/H$. The interruption of flow was better simulated with K = 0 (see Figure 3). Figure 4 represents calculations of flow with rupture of the priming process with K=0 and a constant inflow coefficient $\mu = 0{,}84$ in dependence of relative water level z/H.

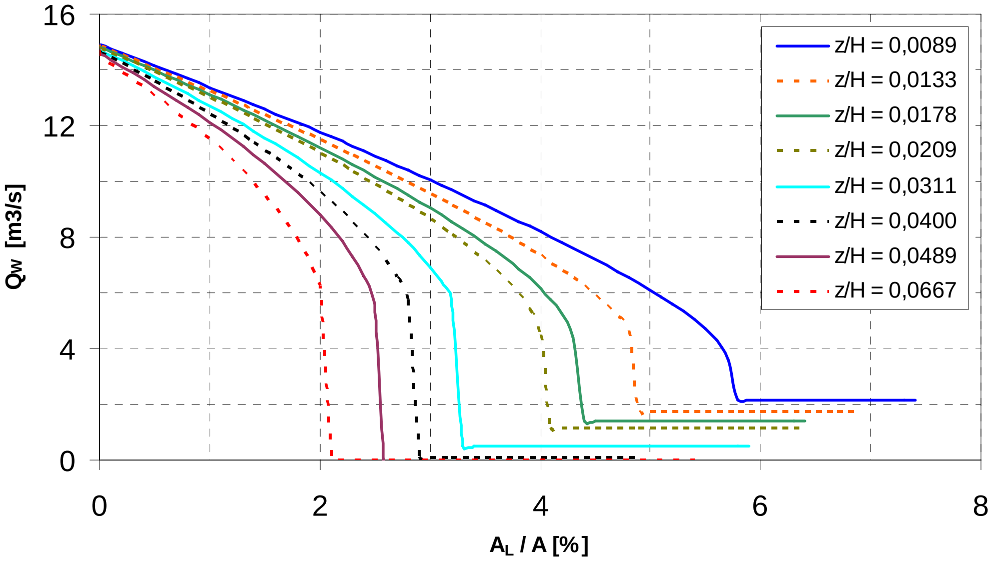

Figure 4: Verification calculation of the rupture process of the siphon model in dependence of the aeration cross section and the dynamic water level of the Oker Dam

3 CONCLUSIONS

When considering the compressibility factor of air, the presented equations for calculating the correct dimensions of an aerated siphon permit an analysis of the influencing factors and provide calculations for the appropriate degree of aeration required for controlling an aerated siphon. A comparison of the results from the model experiments and from the real size data of the Oker Dam siphon underlines the validity of these calculations. The example calculations that we carried out show that the required degree of aeration greatly depends on the height of the water level in the reservoir. When applying the results of the model experiments on to the real life scale, these various influences can also be identified. The various assumptions in the equations such as, for example, the energy loss coefficients due to the uptake of air, the flow coefficient of the air flow and how this is affected by the water flow, or the influence of the water flow turbulence on the rupture process, all highlight the complexities involved when up scaling model results for the application to other types of siphon systems. For such applications additional comparative measurements and / or modelling experiments are indispensable.

REFERENCES

Aigner, D. (2000): Hydraulik der Wasserbehandlungsanlagen und industrieller Prozesse. In *Technische Hydromechanik, Band 4,* Verlag für Bauwesen Berlin 2000,

Horlacher, H.-B.; Dornack, S.; Müller, U. (1995): Hydraulische Untersuchungen für die Heberanlage der Okertalsperre. *Institut für Wasserbau und THM der TU Dresden, Forschungsbericht FO 95/2* vom 23.02.1995, unveröffentlicht

Bollrich, G. (1994): Hydraulische Untersuchung der Heberüberfälle des Ausgleichsbeckens Burgkhammer. *Internationaler Talsperrenkongress in Duban/Südafrika,* November 1994, in IWT Heft 2, Weimar, Oktober 1994

Bollrich, G.; Aigner, D. (2000): Hydraulisches Versuchswesen. *In Technische Hydromechanik, Band 4,* Verlag für Bauwesen Berlin 2000,

Bollrich, G. (2000): Technische Hydromechanik, *Band 1, 5. Auflage, Verlag für Bauwesen* Berlin 2000,

Volkart, P. (1978): Hydraulische Bemessung steiler Kanalisationsleitungen unter Berücksichtigung der Luftmitnahme. *Mitteilungen der Versuchsanstalt für Wasserbau ETH Zürich,* Heft 30, Zürich, 1978

Will, D. W.; Ströhl, H. (1990): Einführung in die Hydraulik und Pneumatik, *5. Auflage, VEB Verlag Technik Berlin,* 1990

Horlacher, H.-B.; Lüdecke, H.-J. (2006): Strömungsberechnung für Rohrsysteme, *2. Auflage, Expert Verlag, Renningen,* 2006

AUTORS:

apl. Prof. Dr.-Ing. habil. **Detlef Aigner**
Prof. Dr.-Ing. habil **Hans.-B. Horlacher**
Institut of Hydraulic Engineering and Applied Hydromechanics
University of Technology Dresden
Helmholtzstrasse 10, 01062 Dresden
Tel.: 0351 463 34397
Fax: 0351 463 37120
detlef.aigner@tu-dresden.de