circular channel, overflow,
brink depth, pipe bend

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OVERFLOW IN A CYLINDRICAL PIPE BEND

The overflow of water in a cylindrical pipe bend discharging with free surface is discussed. The theoretical equation is based on the critical flow, where the energy head is a minimum, and the momentum equation. The solution is only possible with an approximation of the area of the cross section of circular channel. The comparison with experimental results for circular weirs and brink depth in circular channels shows a difference between 15% and 19%, wich can be explained as the influence of the curvature of streamlines.

1. Introduction

During transition from subcritical to supercritical flow critical flow appears, e.g. at drop points, end stills, drops in the river bed or narrows. HAGER (1994) explains necessary and sufficient circumstances concerning this topic.

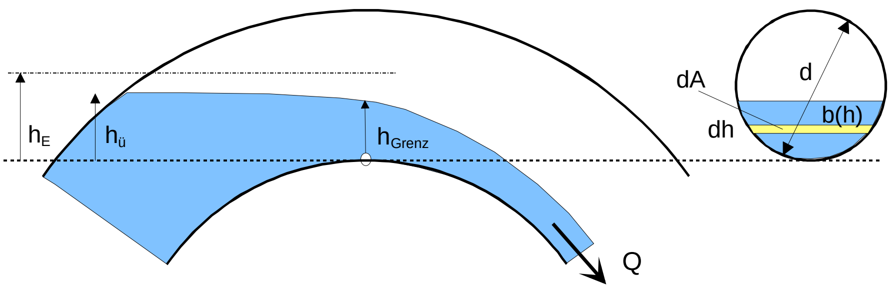

Fig. 1: Overflow in a cylindrical pipe bend

Because the condition of critical flow only appears when the energy level is, relating to the channel bottom, down to a minimum, it is possible to set up a theoretical relation between the discharge and minimum energy head, or the critical water depth with the help of calculating the extreme value of the energy equation of the flow. This possibility may be used to create a theoretical relation between discharge and water depth for various geometries of the cross section. A new relationship between discharge and water level of a turn spillway for the rotating pipe bend, a new control mechanism for waste water channels (CHERUBIM, 1995), was looked for. In literature it was know only (with exception of STAUS) for brink depths and circular weirs above all as empirical solution.

2. Approximation of the area for circular cross-section

For a calculation on a circlar cross section approximate values are necessary, in a way when complete solution are needed. Because of the approximate values the results may show an error, but nevertheless are easier to handle or work with. The real solution compared with the approximate result shows the error and his minimisation. As the overflow equation generally needs a empirical correction, the application of this approximate result may be possible. An example to calculate the cross section area in a partly filled pipe is shown in equation (1), which, especially in the middle of the water level, shows a relatively small error (AIGNER, 1995).

$$A \cong \pi \cdot d^2 \cdot \frac{7}{24} \cdot \left(\frac{h}{d}\right)^{\frac{4}{3}} \tag{1}$$

3. Calculation of critical depth

The relations between the minimum of energy head, the critical head and the discharge can be calculated with the extreme value of the energy head.

$$\frac{dh_E}{dh} = 0 \tag{2}$$

The solution of equation (2) with equation (3)

$$h_E = h + \frac{v^2}{2g} = h + \frac{Q^2}{A^2 \cdot 2g} \tag{3}$$

can be derived from:

$$\frac{Q^2}{g \cdot A^3} \cdot \frac{dA}{dh} = 1 \tag{4}$$

In the case when the second derivation of the equation (3) is greater than zero, the result of equation (4) is a minimum. From equation (3) and (4) follows with the approximation (1):

$$h_{EMin} = \frac{11}{8} \cdot h_{Grenz} \tag{5}$$

With the momentum principle in the end of pipe (Fig.2) and the approximation of eg. (1) the relation between end depth h_e and critical head h_{Grenz} or minimum of energy head h_{Emin} is receired:

$$\frac{h_e}{h_{Grenz}} = 0.742 \tag{6}$$

$$\frac{h_e}{h_{EMin}} = 0.539 \tag{7}$$

The discharge Q, calculated by equations (3) to (7), is a function of one of the three parameters h_e, h_{Grenz}, h_{EMin}:

$$Q = \frac{\pi}{4} \cdot d^{\frac{2}{3}} \cdot \sqrt{g \cdot \frac{49}{48}} \cdot h_{Grenz}^{\frac{11}{6}} = 4.3 \cdot d^{\frac{2}{3}} \cdot h_e^{\frac{11}{6}} = 2.485 \cdot d^{\frac{2}{3}} \cdot h_{Grenz}^{\frac{11}{6}} = 1.386 \cdot d^{\frac{2}{3}} \cdot h_{EMin}^{\frac{11}{6}} \tag{8}$$

or in a non dimensional form:

$$\frac{Q}{\sqrt{g} \cdot d^{2.5}} = 1.371 \cdot \left(\frac{h_e}{d}\right)^{\frac{11}{6}} = 0.794 \cdot \left(\frac{h_{Grenz}}{d}\right)^{\frac{11}{6}} = 0.443 \cdot \left(\frac{h_{EMin}}{d}\right)^{\frac{11}{6}} \tag{9}$$

Correctly is with $A = f(\alpha) = \frac{d^2}{8} \cdot (\alpha - \sin\alpha)$ (1a)

and $\frac{dA}{dh} = f(\alpha) = d \cdot \sin\frac{\alpha}{2}$ (1b)

in a non dimensional form the equation (9a) for the minimum of energy head:

$$\frac{Q}{\sqrt{g} \cdot d^{2.5}} = \frac{A^{\frac{3}{2}}}{d^{2,5} \cdot \sqrt{\frac{dA}{dh}}} = \sqrt{\frac{(\alpha_{Grenz} - \sin\alpha_{Grenz})^3}{512 \cdot \sin\frac{\alpha_{Grenz}}{2}}} \quad (9a)$$

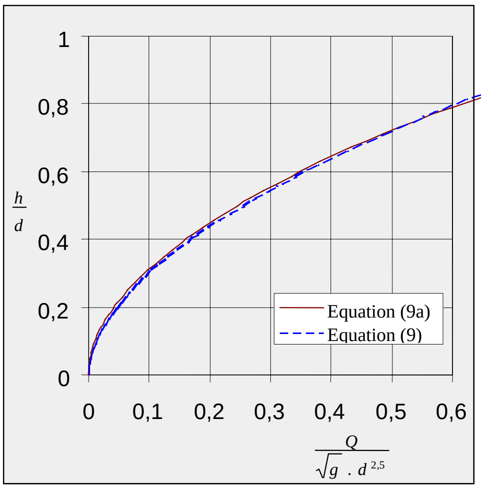

figure : The minimum of energy head. approximately eq. (9) and exact eq. (9a)

4. Comparison with empirical solutions

CHERUBIM (1995):

During the investigation of the overflow of the so called rotating pipe bend, a new control mechanism for waste water channels (Fig.1), overflow results of measurements were analysed. CHERUBIM found the following equation for overflow in a rotating pipe bend in vertical position:

$$Q = C(\beta) \cdot d^{0,68} \cdot h_{EÜ}^{1,82} \quad (10)$$

or non-dimensional:

$$\frac{Q}{\sqrt{g} \cdot d^{2.5}} = 0.516 \cdot \left(\frac{h_E}{d}\right)^{1.82} \quad (11)$$

GREVE (1924)

GREVE analysed sharp edged circular weirs (Fig.2). If the cross section upstream of the weir is extensive, the depth of water h is nearly the energy head ($h \approx h_E$). The exponent of the empirical equation is near 1.88 (1.86 for d=155mm to 1.89 for d=740mm).

$$\frac{Q}{\sqrt{g} \cdot d^{2.5}} = (0.509....0.527) \cdot \left(\frac{h}{d}\right)^{1.88} \quad (12)$$

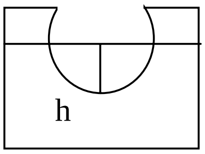

Fig. 2: Circular weir ($h \approx h_E$)

RAJARATHNAM/MURALIDHAR (1991)

Rajarathnam and Muralidhar have investigated the end depth in a cylindrical channel. They found the following function between discharge Q and the water depth at the end of channel h_e:

$$\frac{Q}{\sqrt{g}\cdot d^{2.5}} = 1.54\cdot\left(\frac{h_e}{d}\right)^{1.84} = 0.852\cdot\left(\frac{h_{Grenz}}{d}\right)^{1.84} \qquad (13)$$

The relation of end depth and critical head was a result of many experiments:

$$\frac{h_e}{h_{Grenz}} = 0.725 \qquad (14)$$

$$0 \le \frac{h_e}{d} \le 0.7$$

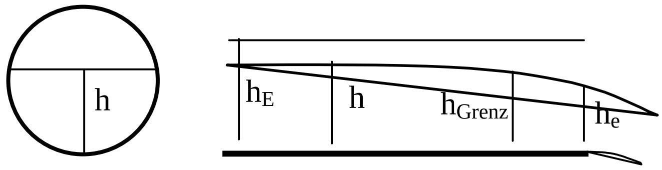

Fig. 3: End depth of pipe

DISKIN, M.H.(1963)

DISKIN shows the dependency between Q and h_e or h_{Grenz} with:

$$\frac{Q}{\sqrt{g}\cdot d^{2.5}} = 1.82\cdot\left(\frac{h_e}{d}\right)^{1.96} = 0.954\cdot\left(\frac{h_{Grenz}}{d}\right)^{1.95} \qquad (15)$$

ADVANI (1963)

In the discussion of the paper of SMITH (1962) Advani specifies the results of RAJARATHNAM und MURALITHAR and other authors. Among other things he shows the result of KING with the following nearly non-dimensional form :

$$\frac{Q}{\sqrt{g}\cdot d^{2.5}} = \frac{1.569}{d^{0.02}}\cdot\left(\frac{h_e}{d}\right)^{1.88} \quad \text{(d in m)} \qquad (16)$$

SMITH (1962)

Smith has stimulated a vivid discussion with his equations for discharge measurement at the end of pipe. He found equation (17) with the momentum principle at the end of pipes. The non-dimensional form of his equation is:

$$\frac{Q}{\sqrt{g}\cdot d^{2.5}} = 1.463\cdot\left(\frac{h_e}{d}\right)^{1.84} \qquad (17)$$

HAGER (1995)

HAGER explained this subject of outflow at the end of pipes already from the beginning in 1920 (MCAULIFFE), followed by VANLEER (1922) and ROHWER (1943). This measurement-method by VANLEER called "California-method" was said to reach an error less than 5%.

RAMPONI (1936) in STAUS (1937)

RAMPONI has investigated a sharp edged circular weir (Fig.2). He found a different equation for the overflow with a weir coefficient μ as function of the relative head ($h \approx h_E$).

$$\frac{Q}{\sqrt{g}\cdot d^{2.5}} = \mu \cdot \left(1.022 \cdot \left(\frac{h}{d}\right)^{1.975} - 0.269 \cdot \left(\frac{h}{d}\right)^{3.78}\right) \qquad (18)$$

with $\mu = 0.555 + \frac{1}{110 \cdot \frac{h}{d}} + 0.041 \cdot \frac{h}{d}$ after STAUS (1937).

5. Summary

Starting off with the beginnings of discharge measurement at the end of pipes nowadays the dependency between discharge and energy head, critical head or end depth can be specified with the given equations. The theoretical solution, which was derived with the help of the approximation of the cross section area, shows a difference to the empirical solutions of CHERUBIM, GREVE and others. The theoretical solution thereby give a 15% to 19% smaller value then the empirical. The multiplication of the right hand side of equation (9) with 1.165 shows, with errors of 4 or 5%, that it is possible to calculate the discharge as function of end depth, critical head or energy head, when the conditions given by the different authors are taken into consideration.

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