Fuzzy probabilistic structural analysis considering fuzzy random functions

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ABSTRACT: The system behavior may only be realistically assessed provided all input data are appropriately described and a realistic computational model is implemented. In the paper a new concept of modeling is presented, based on the theory of fuzzy random functions. By this means, a super-ordinate uncertainty model is made available which includes the models developed so far, based on real random values and fuzzy values as special cases. For the analysis of a structure with the aid of a crisp (or uncertain) algorithm and with fuzzy random functions (or random functions) as input values as well as fuzzy values as model parameters, a fuzzy probabilistic structural analysis is introduced.

1 INTRODUCTION

The realistic analysis of structures requires reliable (input) data as well as suitably-matched computational models; as a rule the data and the model contain uncertainty. In contrast to deterministic structural analysis, fuzzy probabilistic structural analysis takes account of this data and model uncertainty.

The geometrical, material and loading data required for structural analysis are more or less characterized by uncertainty. It is necessary to appropriately take this uncertainty into consideration.

If an event (regarding its occurrence), as a random result of a test, may be observed as a crisp value on an almost unlimited number of occasions under constant boundary conditions, this concerns a stochastic uncertainty. The uncertainty characteristic randomness is assigned to this stochastic uncertainty. If the boundary conditions are (apparently) subject to arbitrary fluctuations, a comprehensive system overview is lacking, the number of observations are only available to a limited extend, or the sample elements are of doubtful accuracy (non-precise), an information deficit exists. The outcome of this is a gap between the mathematical quality requirements of data if using stochastic methods and the real available non-precise data. The data do not satisfy real-valued probability laws. In fact the data may be quantified by imprecise probability (see e.g. Viertl 1996).

The impreciseness results from informal uncertainty in terms of non-precise recognition of data or statistical inference (determining stochastic input parameters, such as expected values, variances and probability distribution functions). Here, this informal uncertainty is described by the uncertainty characteristic fuzziness and mathematically quantified on the basis of the fuzzy set theory. The uncertainty consisting of randomness and fuzziness is summarized in the characteristic fuzzy randomness. The fuzzy random data are assessed with the aid of the uncertain measure *fuzzy probability*.

The uncertainty characteristic fuzzy randomness includes both randomness and fuzziness as special cases. If data only show random properties, fuzziness is quantified by zero, i.e. a real-valued random variable is used. Non-precise data without random properties are quantified by fuzzy values. Fuzzy randomness is a generalized uncertainty model because it permits to consider randomness and fuzziness simultaneously.

Data can also be assessed with the aid of the uncertain measure interval probability (Sarveswaran, Smith & Blockley 1998), which may be interpreted as a special case of fuzzy probability. The impreciseness is quantified with non-assessed intervals. This intervals are special fuzzy values, whose membership functions can only become zero and unity.

Uncertain data (e.g. for material parameters, loading or boundary conditions) may be characterized by fuzzy random fluctuations, which depend on external conditions. External conditions include, for example, time τ, the spatial coordinates $\underline{\theta} = \{\theta_1, \theta_2, \theta_3\}$, air pressure or temperature, which are lumped together in the parameter vector $\underline{t} = \{\tau, \underline{\theta}, ...\}$. If this relationship can be formulated in terms of a fuzzy random function, one fuzzy random value is uniquely assigned to each realization of the parameter vector $\underline{t}$.

In order to take account of fuzzy random functions including the discussed special cases the fuzzy probabilistic structural analysis is developed. This comprehensive analysis concept is formulated as a further development of introduced probabilistic and fuzzy probabilistic approaches.

2 FUZZY RANDOM FUNCTIONS

The underlying mathematical approach of this work is founded on the definitions of fuzzy random variables assembled in Möller, Beer, Graf & Sickert 2001b that are based on Kwakernaak 1978. The definition of fuzzy random functions for n-dimensional parameter vectors $\underline{t}$ is an enhancement of the work of Guangyuan Wang & Yue Zhang 1992 concerning one-dimensional t.

The probabiliy space $[\Omega, \mathfrak{S}, P]$ is extended by the dimension of fuzziness on the basis of the KOLMOGOROW's axiomatic probability concept; the uncertain measure probability remains defined over the n-dimensional EUCLIDian space $\mathbb{R}^n$.

Thereby Ω designates the space of the elementary events and $\mathfrak{S}$ denote a σ-algebra in $\mathbb{R}^n$. $\mathbf{T} \in \mathbb{R}^m$ designates the m-dimensional space of the parameters $\underline{t} \in \mathbf{T}$. It establishes the fundamental set of values that are possible realizations of the fuzzy random function.

A fuzzy random function $\tilde{\underline{X}}(\underline{t})$ is then the fuzzy result of the mapping $\mathbf{T}\times\Omega$ onto $\mathbf{F}(\mathbb{R}^n)$, whereby $\mathbf{F}(\mathbb{R}^n)$ denotes the set of all fuzzy values in $\mathbb{R}^n$. It is a family of fuzzy random variables $\tilde{\underline{X}}$ for fixed $\underline{t} = \underline{t}_k$ in the extended probability space $[\Omega, \mathfrak{S}, P]$ explained above.

$$\tilde{\underline{X}}(\underline{t}) = \{\tilde{\underline{X}}(\underline{t}_k, \omega), \underline{t} \in \mathbf{T}, \omega \in \mathbf{\Omega}, k = 1, 2, 3, ...\} \qquad (1)$$

An n-dimensional fuzzy function, whose function values are fuzzy values (see Möller, Beer, Graf & Sickert 2001a), is assigned to each (crisp) elementary event $(\omega, \underline{t})$ of the fuzzy random function. Thus the fuzzy function is a realization of the fuzzy random function. In Fig. 1 three realizations of an one-dimensional fuzzy random function are displayed exemplarily.

If the realization $\underline{x}(\underline{t})$ of a real random function $\underline{X}(\underline{t})$ as well as the fuzzy realization $\tilde{\underline{x}}(\underline{t})$ of a fuzzy random function $\tilde{\underline{X}}(\underline{t})$ may be assigned to an elementary event ω, and if $\underline{x}(\underline{t}) \in \tilde{\underline{x}}(\underline{t}) \;\forall\; \underline{t} \in \underline{\mathbf{T}}$ holds, this means that $\underline{x}(\underline{t})$ is contained in $\tilde{\underline{x}}(\underline{t})$. If for all elementary events $\omega \in \Omega$ the $\underline{x}(\underline{t})$ are contained in the $\tilde{\underline{x}}(\underline{t})$, the $\underline{x}(\underline{t})$ then constitute an original function $\underline{X}(\underline{t})$ of the fuzzy random function $\tilde{\underline{X}}(\underline{t})$. Each real random function $\underline{X}(\underline{t})$ without fuzziness that is completely contained in $\tilde{\underline{X}}(\underline{t})$ is thus an original function of the fuzzy random function $\tilde{\underline{X}}(\underline{t})$. Provided that all original functions are known, a fuzzy random function is the fuzzy set of its original functions contained in $\tilde{\underline{X}}(\underline{t})$. Realizations of the real random functions (original function) are reffered to as trajectories (crisp exemplar function).

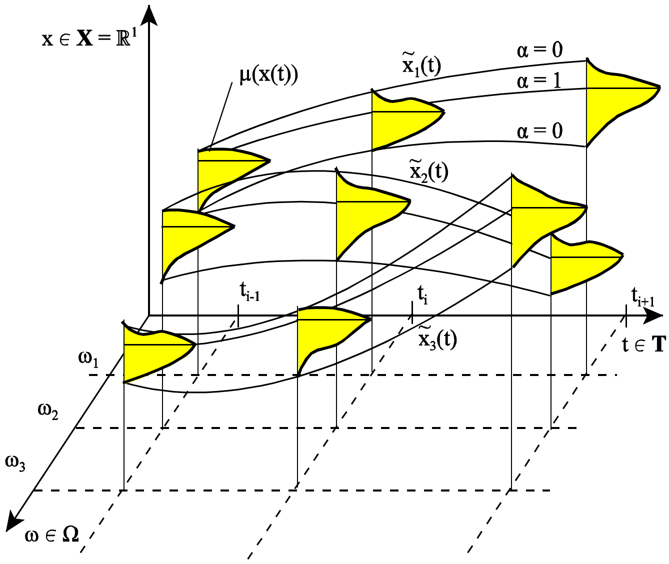

Figure 1. Realizations of a one-dimensional fuzzy random function

Each fuzzy random function $\tilde{\underline{X}}(\underline{t})$ contains at least one real random function $\underline{X}(\underline{t})$ as an original function of $\tilde{\underline{X}}(\underline{t})$. Thus each fuzzy random function $\tilde{\underline{X}}(\underline{t})$ that possesses precisely one original is an real random function $\underline{X}(\underline{t})$. The description of fuzzy random functions by means of their original functions ensures that real random functions are contained in fuzzy random function as a special case.

With the aid of α-discretization a fuzzy random function may be formulated as a set of α-level sets of real random functions (original functions)

$$\begin{aligned}\tilde{\underline{X}}(\underline{t}) = \{&\underline{X}_\alpha(\underline{t});\; \mu(\underline{X}_\alpha(\underline{t})) \\ &|\; \underline{X}_\alpha(\underline{t}) = [\underline{X}_{\min,\alpha}(\underline{t});\; \underline{X}_{\max,\alpha}(\underline{t})], \\ &\mu(\underline{X}_\alpha(\underline{t})) = \alpha, \quad \forall \alpha \in (0,1]\}\end{aligned} \qquad (2)$$

For a fuzzy random function that is solely dependent on spatial coordinates $\underline{\theta}$ the term *fuzzy random field* is adopted. In the case of time-dependency, the term *fuzzy random process* is adopted.

The properties of fuzzy random functions regarding their randomness may be derived from the theory of random functions. A fuzzy random function is e.g. strictly stationary, if the moments of all orders of the fuzzy probability distribution function are invariant relative to a displacement in the vector $\underline{t}$.

For the fuzzy random function, fuzzy probability distribution functions $\tilde{F}_t(\underline{x})$ may be generated as a fuzzy set of the probability distribution functions $F_t(\underline{x})$ of the original functions with the membership values $\mu(F_t(\underline{x}))$. The quantification of fuzziness by fuzzy parameters leads to the description of the fuzzy probability distribution function $\tilde{F}_t(\underline{x})$ of $\underline{\tilde{X}}$ as a function of the fuzzy bunch parameter vector $\underline{\tilde{s}}$.

$$\tilde{F}_t(\underline{x}) = F_t(\underline{\tilde{s}}, \underline{x}) \tag{3}$$

Special bunch parameters are e.g. the fuzzy mean value of a fuzzy random field characterized by a constant mean value

$$\tilde{s} = \tilde{m}_x(\underline{\theta}) = E\left[\tilde{X}(\underline{\theta})\right] = \int_{x=-\infty}^{x=+\infty} x \cdot \tilde{f}_{\underline{\theta}}(x)\, dx \tag{4}$$

or the parameters of the fuzzy probability distribution function of an homogeneous isotropic fuzzy random field that is GUMBEL distributed

$$F_\theta(\underline{\tilde{s}}, x) = \exp\left(-\exp\left(-\tilde{s}_1\,(x - \tilde{s}_2)\right)\right) \tag{5}$$

The uncertain proportion of normal distribution (ND) and logarithmic normal distribution (LND) of a mixed probability distribution function may also be quantified by a fuzzy bunch parameter.

$$F_\theta(\tilde{s}, x) = \tilde{s} \cdot F_\theta^{ND}(x) + (1 - \tilde{s}) \cdot F_\theta^{LND}(x) \tag{6}$$

In the special case of homogeneous isotropic fuzzy random fields the linear dependency of the fuzzy random variables may be described using fuzzy correlation functions. Fuzzy correlation functions $\tilde{k}_X(L)$ are mostly fuzzy functions selected without experimental verification for the mathematical quantification of the fuzzy correlation $\tilde{R}_{XX}(L)$. An exponential and a linear shape function are shown in Fig. 2.

3 FUZZY PROBABILISTIC STRUCTURAL ANALYSIS

The aim of fuzzy probabilistic structural analysis is to map fuzzy random input data $\underline{\tilde{X}}$ onto fuzzy random structural responses $\underline{\tilde{Z}}$ (e.g. displacements, internal forces, strains or stresses).

$$\underline{\tilde{X}}(\underline{t}) \rightarrow \underline{\tilde{Z}}(\underline{t}) \tag{7}$$

One component of the mapping operator is a deterministic linear or nonlinear, static or dynamic structural analysis, which is referred to as deterministic fundamental solution. The special combination of the uncertainty model "fuzzy randomness" with a deterministic algorithm based on the Finite Element Method (FEM) leads to *Fuzzy Stochastic Finite Element Method (FSFEM).*

The abstract procedure of the fuzzy probabilistic structural analysis is shown in Fig. 4. Fuzzy random input parameters are quantified by fuzzy random functions. Depending on the kind of parameter vector $\underline{t}$ influencing the fuzzy random fluctuations the special cases of fuzzy random functions may be derived, e.g. fuzzy random variables (independent of $\underline{t}$), fuzzy random fields (only depending on the position vector $\underline{\theta} = \underline{t}$) or fuzzy random processes (only depending on the time $\tau = t$). All further uncertain parameters are put in as deterministic values or fuzzy values.

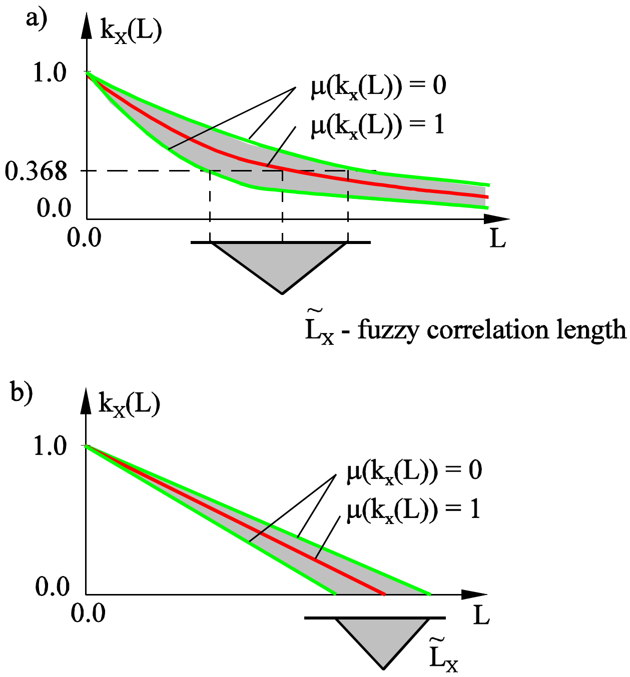

Figure 2. Fuzzy correlation functions

The fuzziness of the fuzzy random functions $\underline{\tilde{X}}(\underline{t})$ is described by way of fuzzy bunch parameters $\underline{\tilde{s}}$. With the aid of α-discretization the fuzzy bunch parameters $\underline{\tilde{s}}$ are discretized by crisp α-level sets (intervals) of each predetermined membership level α.

The mapping operator of the fuzzy probabilistic structural analysis consists of a three-part analysis algorithm (Fig. 3) and includes fuzzy analysis, probabilistic structural analysis and deterministic structural analysis (deterministic fundamental solution).

The aim of fuzzy analysis (outer loop) is to map fuzzy input values (bunch parameters and input parameters) onto result values with the aid of an analysis algorithm. The results $\tilde{z}_j$ are also fuzzy values. They may be computed from the fuzzy input values by means of the extension principle in combination with the Cartesian product between uncertain sets (see e.g. Zadeh 1965 or Beer 2002). However, the extension principle is hardly practicable in the case of complex mapping operators, as its application requires discretization of the support of the fuzzy input sets - e.g. using a point mesh. This leads to numerical problems. Alternative procedures which exploit the special properties of the mapping operator or additional information concerning the mapping are suggested e.g. in Bonarini & Bontempi 1994. Here the *α-level optimization* according to Möller, Graf & Beer 2000 is applied which permits the use of mapping operators without special properties.

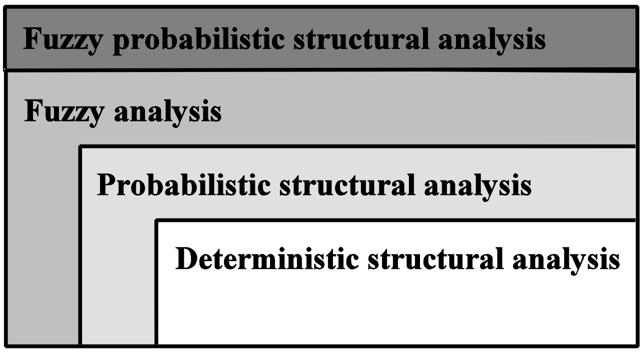

Figure 3. Three-part analysis algorithm of fuzzy probabilistic structural analysis

All fuzzy input values are discretized using the same number of α-levels α_k, k = 1, ..., r (Fig. 4 ②). For each fuzzy input value $\tilde{s}_i = \tilde{A}_i$ on the level α_k the α-level set A_{i,α_k} is then assigned to $\tilde{s}_i$, and all A_{i,α_k} form the crisp subspace $\underline{S}_{\alpha_k}$ (Fig. 4 ③). With the aid of the mapping operator $\underline{z} = f(s_1; \ldots ; s_n)$ it is possible to compute elements of the α-level sets B_{j,α_k} of the fuzzy bunch parameters $\tilde{z}_j = \tilde{B}_j$, j = 1, ..., m of the result values $\underline{\tilde{Z}}(t)$ on the α-level α_k (Fig. 4 ⑧). The mapping of all elements of $\underline{S}_{\alpha_k}$ yields the crisp subspace $\underline{Z}_{\alpha_k}$ of the z-space.

Once the largest element $z_{j,\alpha_k r}$ and the smallest element $z_{j,\alpha_k l}$ of the α-level set B_{j,α_k} have been found, two points of the membership function $\mu(z_j) = \mu_{B_j}(z_j)$ are known (Fig. 4 ⑧). In the case of convex fuzzy result values the $\mu(z_j)$ are thus completely described. The determination of $z_{j,\alpha_k r}$ and $z_{j,\alpha_k l}$ replaces the max-min operator of the extension principle. The search for the smallest and largest elements may be formulated as an optimization problem. The objective functions

$$z_j = f_j(s_1; \ldots ; s_n) \Rightarrow \text{Max} \mid (s_1; \ldots ; s_n) \in \underline{S}_{\alpha_k} \tag{8}$$

$$z_j = f_j(s_1; \ldots ; s_n) \Rightarrow \text{Min} \mid (s_1; \ldots ; s_n) \in \underline{S}_{\alpha_k} \tag{9}$$

must be satisfied. The requirements $(s_1; \ldots; s_n) \in \underline{S}_{\alpha_k}$ represent the restrictions of the optimization problem.

Eqns. (8) and (9) are satisfied by the optimum points $\underline{s}_{opt}$. For each fuzzy result value precisely two optimum points in the crisp subspace $\underline{S}_{\alpha_k}$ belong to each α-level α_k. The optimization task according to Eqns. (8) and (9) for all α-levels α_k and all fuzzy bunch parameters $\tilde{z}_j$ of the result values $\underline{\tilde{Z}}(t)$ is referred to as *α-level optimization.*. In order to solve the α-level optimization problem special properties of the mapping operator $\underline{z} = f(s_1; \ldots ; s_n)$ may be exploited; these include *uniqueness, biuniqueness*, *continuity, monotonicity* and *dimensionality of the s-space and z-space*.

If the mapping operator possesses no special properties, the optimum points $\underline{s}_{opt}$ are located arbitrarily in $\underline{S}_{\alpha_k}$; otherwise the search for the $\underline{s}_{opt}$ may be limited to parts of $\underline{S}_{\alpha_k}$ - e.g. on the "boundary". If

a) every crisp subspace $\underline{S}_{\alpha_k}$ is coherent and
b) the mapping operator is continuous and unique,

the fuzzy values $\tilde{z}_j$ are then convex uncertain sets. If no interaction exists between the fuzzy input values $\tilde{s}_i$ condition a) is satisfied when all $\tilde{A}_i = \tilde{s}_i$ are convex uncertain sets. If condition b) is not complied with, the α-level optimization yields envelope curves of the actual membership functions of the $\tilde{z}_j$. The applied optimization strategy is explained in Möller, Graf & Beer 2000.

Each element of the fuzzy set $\tilde{\underline{s}}$ determines one original function (real random function) of every fuzzy random function and one original (deterministic function) of the fuzzy correlation function (Fig. 4 ④). These original functions are mapped onto original functions $Z(\underline{t})$ of the fuzzy random result values $\tilde{Z}_r(\underline{t})$ by means of an efficient probabilistic analysis algorithm (middle loop in Fig. 3 and Fig. 4 ⑤). The deterministic (nonlinear) static or dynamic structural analysis is performed in the inner loop (Fig. 3 and Fig. 4 ⑥).

Monte Carlo simulation is an efficient probabilistic analysis algorithm (see e.g. Schuëller 2001). It represents a suitable universal tool when applying complex nonlinear models in deterministic structural analysis. Both the spectral representation of the real random functions for random fields and the special methods for the analysis of MARKOV chains for random processes may be applied.
The result of one Monte Carlo simulation is a sample according to one original function $Z(\underline{t})$ of the fuzzy random structural response $\tilde{Z}_r(\underline{t})$ on the membership level $\mu(Z(\underline{t})) = \alpha_k$ (Fig. 4 ⑦). The assigned elements z_{j,α_k} of the fuzzy bunch parameters $\tilde{z}_j$ (e.g. fuzzy mean value, fuzzy variance or fuzzy quantils) of the fuzzy random result values $\tilde{Z}_r$ may be obtained by statistical evaluation of the sample. The assigned fuzzy probability distribution function may be determined directly from empirical distribution function or approximated applying the methods of statistical inference and test theory.

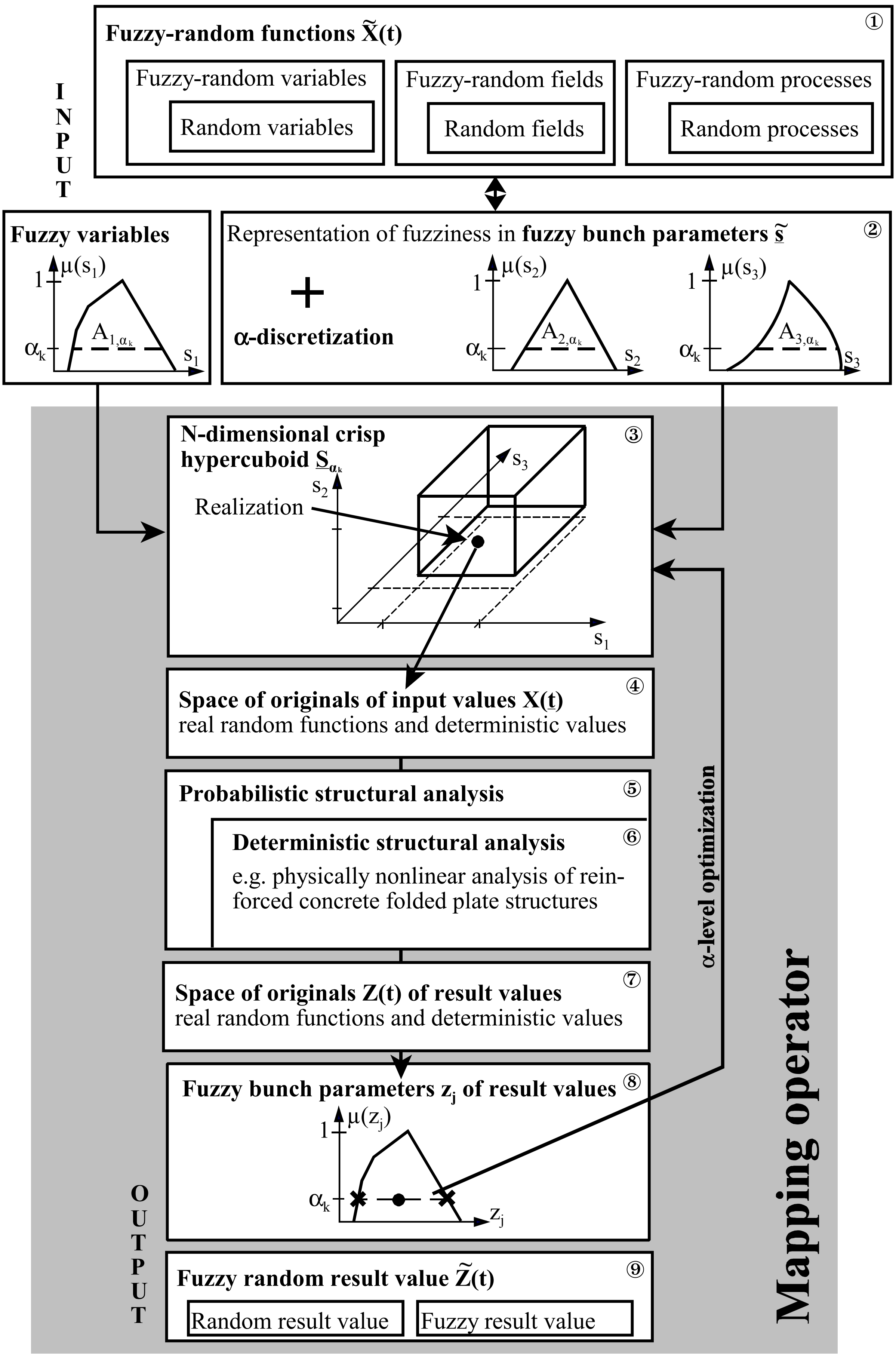

Figure 4. Scheme of the fuzzy probabilistic structural analysis

The special case, that only fuzzy values $\underline{\tilde{s}} = \underline{\tilde{x}}$ have to be taken into account as input parameters of a structural analysis, is also implemented in the algorithm described above. Then the analysis of the probabilistic algorithms is omitted. Using the α-level optimization in combination with the deterministic fundamental solution, the fuzzy result values $\underline{\tilde{z}}$ concerning the structural response may be determined (Fig. 4 ⑨).

Fuzzy probability distribution functions for fuzzy random result values may be approximated on the basis of the set of fuzzy quantil values. The use of the computed fuzzy bunch parameters in the equation of an assumed (tested) probability distribution is also possible.

4 EXAMPLE

The fuzzy probabilistic structural analysis approach is demonstrated by way of an example. The uniaxial plate (Fig. 5) is analyzed under consideration of the governing nonlinearities of reinforced concrete. The physical nonlinear deterministic fundamental solution is represented by the FE program FALT-FEM for the calculation of folded plates on the basis of mixed hybrid finite elements with assumed stress distribution (see Möller, Graf & Kluger 1997). Endochronic material laws are applied to concrete and reinforcement steel. Tensile cracks in the concrete are accounted for in each element on a layer-to-layer basis according to the concept of smeared fixed cracks. Uncertain input parameters are both the time-dependent fuzzy random fluctuations of the distributed load and the fuzzy random fluctuating conrete compressive strength in the whole plate. Each finite element consists of 12 concrete layers (of 1mm thickness each) and two uniaxial smeared reinforcement layers. The loading process (Fig. 6) consists of the dead weight, a fuzzy random distributed load and a nodal load P = 1 kN in the center of the plate.

Investigation 1

The investigation aimes on the determination of the fuzzy random crack state under the service load.

The distributed load $\tilde{p} \cdot \nu(\tau)$ is modeled by a fuzzy random process. The factor $\tilde{p}$ is considered to be a discrete GUMBEL distributed fuzzy random variable with a fuzzy expected value $\tilde{E}[p]$ (Fig. 6) and a standard deviation $\sigma_p = 0.1$ kN/m².

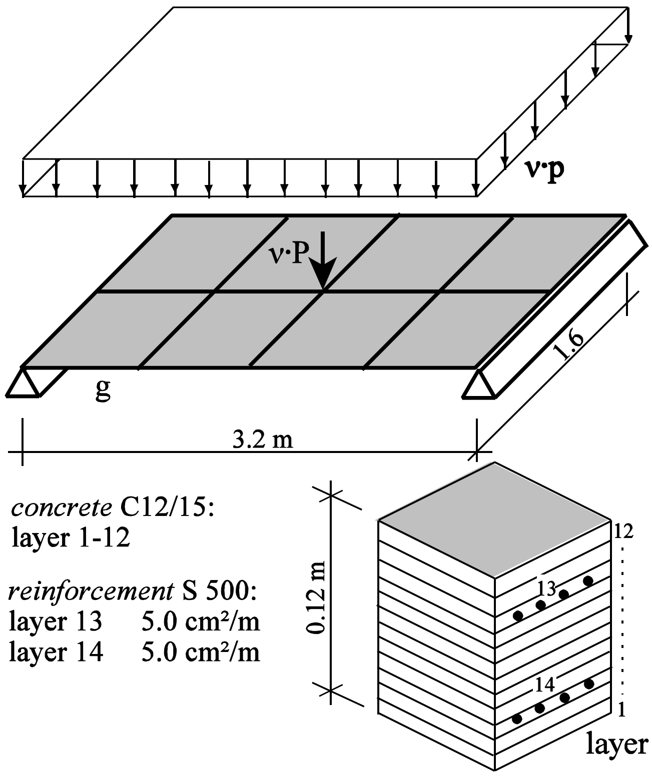

Figure 5. Geometry, FE model

$$F(\underline{\tilde{s}}, p) = \exp\left(-\exp\left(-\tilde{s}_1 (p - \tilde{s}_2)\right)\right) \quad (10)$$

$$\tilde{s}_1 = s_1 = 1.28255 / \sigma_p \quad (11)$$

$$\tilde{s}_2 = \tilde{E}(p) - 0.577216 / \tilde{s}_1 \quad (12)$$

The concrete compressive strength is modeled by a perfectly correlated GAUSS normal distributed random field with the expected value E[β] = 20 N/mm² and a coefficient of variation v = 0.10. The concrete tensile strength is calculated from the compressive strength with an endochronic concrete material law and under consideration of the strain velocity. Therefore it is a random variable as well. Because of the perfect correlation the introduction of a random variable that mirrors the random fluctuations of all elements at once is sufficient.

The probabilistic structural analysis is carried out by Monte-Carlo simulation. A sample consisting of 100 sample elements is calculated from each element of the fuzzy expected value $\tilde{E}[p]$ (fuzzy bunch parameters). A trajectory of the loading process and a realization of the concrete compressive strength are determined for each sample element. The load is increased incrementally up to τ = 2000 d and the crack state is iteratively computed in every increment.

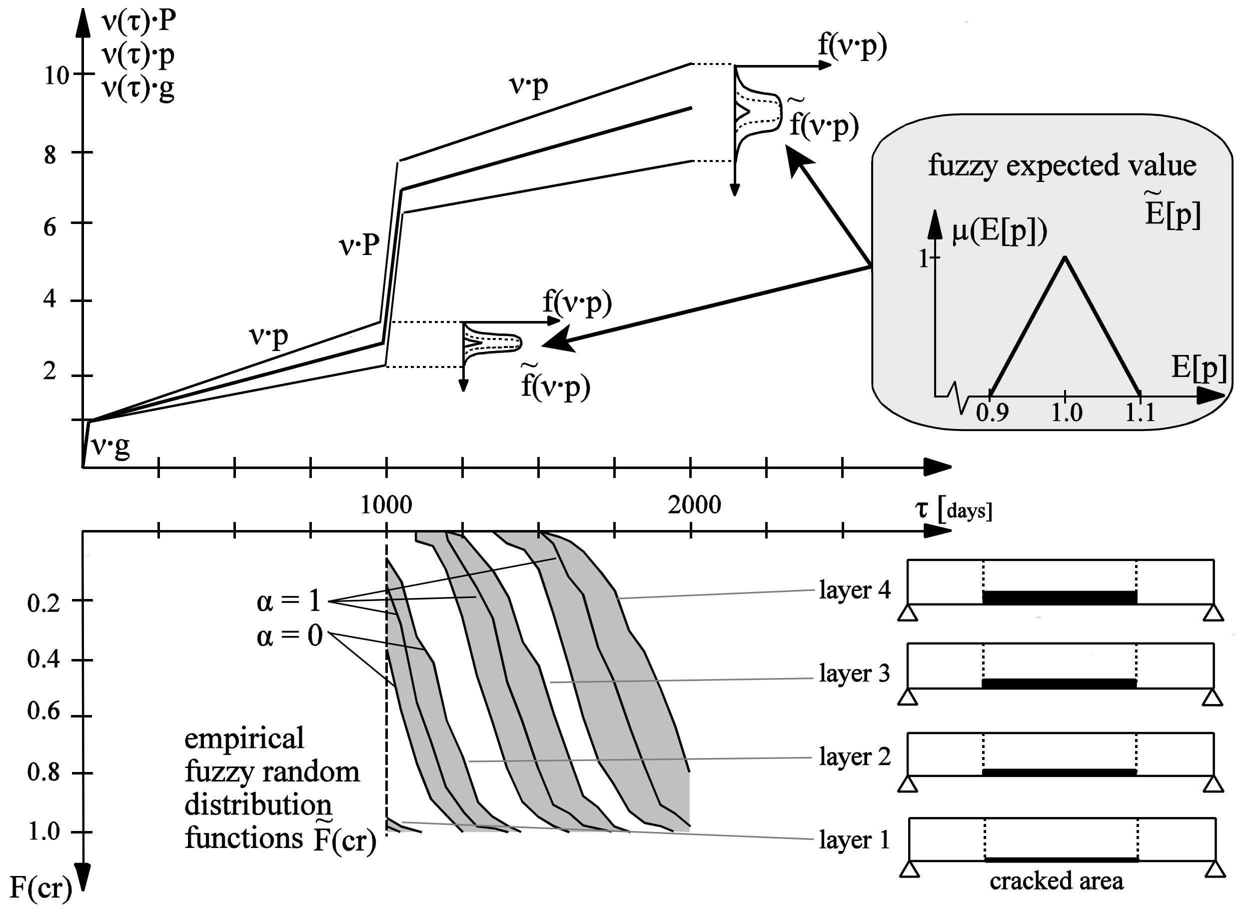

Figure 6. Fuzzy random load process, fuzzy random crack state

The empirical fuzzy random distribution functions $\tilde{F}(cr)$ for the points in time when the first cracks occur are shown in Fig. 6. The lower three layers were cracked in one direction for each simulated realization. The first cracks in layer 1 occured predominately when applying the point-load. Layer 4 remained uncracked for some realizations. In consequence of the multiplicative connection of $\nu(\tau)$ and $\tilde{p}$ the uncertainty of the loading process as well as the uncertainty of the crack state increase with time. This can be concluded from the growing distance between the empirical fuzzy random distribution functions of the decisive original functions for $\alpha = 0$. A probabilistic analysis, which does not consider the fuzziness of the expected value $\tilde{E}[p]$, yields only one empirical distribution function for each layer (marked by $\alpha = 1$ in Fig. 6).

Investigation 2

Uncertain input parameters are the fuzzy random loading process according to Fig. 6 and the concrete compressive strength, as well. Now the latter is modeled by a partially correlated fuzzy random field in extension of investigation 1. The field is discretized with fuzzy random variables in the centroids of the eight finite elements. This discretization method (midpoint method; Schuëller 2001) is chosen considering the special properties of the applied nonlinear FE-algorithm. The correlation of the fuzzy random variables is computed by evaluating the correlation function in Fig. 2b with a fuzzy correlation length $\tilde{L}_x$ = <2, 4, 10>[m] (fuzzy triangular number). Additionally, the position of the reinforcement in the tensile zone is modeled by a fuzzy variable, i.e. without randomness. Thus the space of the fuzzy bunch parameters possesses three dimensions.

The fuzzy random displacement $\tilde{v}$ in the plate center θ_m after $\tau = 2000$ d is chosen as fuzzy random result value.

$$\tilde{Z} = \tilde{v}\,(\underline{\theta}_m, \tau = 2000 \text{ d}) \qquad (13)$$

In Fig. 7 the fuzzy bunch parameter $\tilde{\bar{v}}$ (fuzzy mean value) of $\tilde{v}$ is shown. The fuzzy empirical distribution function of v is displayed in Fig. 8.

5 CONCLUSIONS

For the analysis of a structure with the aid of a crisp (or uncertain) algorithm and with fuzzy random functions (or random functions) as input values, a fuzzy probabilistic structural analysis is introduced. The results $\tilde{Z}_r(\underline{t}) \in \underline{\tilde{Z}}(\underline{t})$ of the fuzzy probabilistic structural analysis are computed as fuzzy random functions of the structural responses.

The fuzzy probabilistic structural analysis permits the simultaneous consideration of different types of uncertainty: randomness, fuzzy randomness and fuzziness.

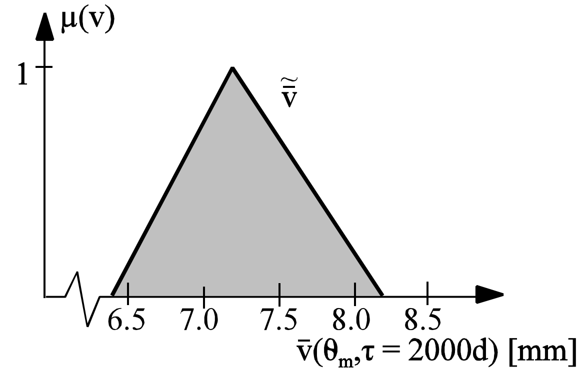

Figure 7. Fuzzy mean value of the fuzzy random displacement

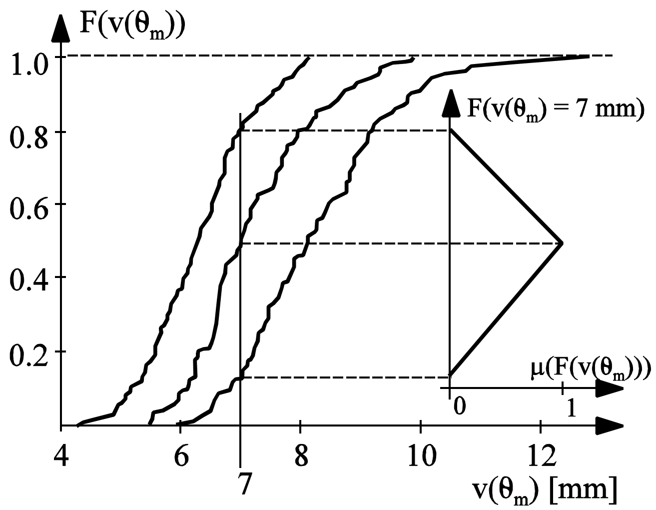

Figure 8. Fuzzy empirical distribution function of the fuzzy random displacement

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