LECTURE COURSE

Relativistic Microarcsecond Astrometry

Compact course: 20 hours

General information:

General relativity theory plays a very important role in modern astronomy. In high precision astrometry this theory is considered as a standard theory to be used for processing and interpreting modern high-accuracy observations. A basic knowledge of general relativity is thus indispensable for any student in this field. This course is intended for post-graduate students in astronomy (especially for those who specializes in astrometry and celestial mechanics, but could be very useful for all students who can profit from understanding general relativity). The aim of this course is to give the students basic knowledge of the theory and to show the most important applications of general relativity theory in the field of astrometry. Prerequisites for this course are general mathematical courses (algebra, mathematical analysis, differential equations, numerical analysis and mathematical physics), as well as the courses in theoretical mechanics, celestial mechanics and astrometry.

Introduction (1 h)

Subject of relativistic astrometry and celestial mechanics. History of relativistic astrometry and celestial mechanics. Perihelion advance of Mercury. Suggested explanations of the excessive advance in the framework of Newtonian mechanics. History of Special Relativity Theory (SRT). History of General Relativity Theory (GRT). Classical tests of GRT. Development of GRT in the 20s–80s. Its status in astronomy: GRT as a foundation of astrometry and celestial mechanics. What and how we are going to learn in this course. Recommended literature to the lectures.

I Essential Tensor Calculus and Riemannian Geometry (5 h)

§ 1 Tensors in Euclidean space (2.7 h)

Vectors in Euclidean space. Covariant and contravariant components of vectors. Metric tensor. General definition of a tensor. Basic tensor operations. Differentiation of tensor fields: partial and covariant derivatives. Christoffel symbols of the first and second kinds. Absolute derivative. Parallel transport along a curve.

§ 2 Tensors in Riemannian space (1.5 h)

Definition of a Riemannian space. Metric tensor of a Riemannian space. Tangent space. Geodesics. Curvature tensor, Ricci tensor, scalar curvature.

§ 3 Pseudo-Euclidean and pseudo-Riemannian spaces (0.8 h)

The metric as quadratic form. Signature of metric. Euclidean and pseudo-Euclidean spaces. Riemannian and pseudo-Riemannian spaces. Elements of pseudo-Euclidean and pseudo-Riemannian geometry.

II Foundations of General Relativity Theory (6 h)

§ 1 Newtonian mechanics and Newtonian theory of gravitation (0.5 h)

Space and time in Newtonian physics: uniformity of space and time, isotropy of space, Euclidean properties of space, absolute character of time. Inertial reference systems of Newtonian mechanics and Galilean principle of relativity. Relativity of space in Newtonian mechanics. Weak equivalence principle. Newtonian gravitational field equations. Infinite velocity of propagation of gravitational field in Newtonian physics.

§ 2 Basic ideas of Special Relativity Theory (1.5 h)

Definition of inertial reference systems in SRT. Special principle of relativity. Interval and its invariance in all inertial reference systems, pseudo-Euclidean properties of event space in SRT. Minkowski metric. Lorentz transformations. Some consequences of the Lorentz transformations. World line of a particle. Isotropic (null), spacelike and timelike intervals and geodesics. Light cone. Absolute future, absolute past and absolute distant events. 4-velocity of a particle and its relationship to the 3-dimensional spatial velocity. Energy-momentum tensor in SRT. Conservation laws in SRT.

§ 3 Basic ideas of General Relativity Theory (2.5 h)

Unique properties of gravitation as a force: Einstein's elevator. Einsteinian principle of equivalence. Relationship between Einsteinian and weak equivalence principles. Locally inertial reference systems. Pseudo-Riemannian properties of event space in GRT. Quasi-Cartesian coordinates. Field equations of GRT and how to "derive" them. Cosmological constant. Gauge conditions and their role to solve the GRT field equations. Harmonic reference systems. Equations of motion of test particles in GRT: geodesic principle and its relationship to the Einsteinian principle of equivalence. Light propagation in GRT: the limit of geometrical optics. Exact solutions of the Einstein equations: Schwarzschild and Kerr solutions.

§ 4 Observations in General Relativity (1.5 h)

Observable and non-observable (coordinate dependent) quantities. Modeling of observations in SRT. Test observers. Proper time. Locally inertial reference systems of an observer: tetrad formalism. Physical meaning of a tetrad. Manipulations with tetrad indices. Equations defining the vectors of a tetrad. The use of tetrads for calculating interval of observable (proper) time and observable distance between two infinitesimally close events. Laws for the change of the vectors of a tetrad for a moving observer. Tetrad as a coordinate basis of the local reference system of an observer. Fermi-Walker transport. Dynamically and kinematically nonrotating tetrads.

III Post-Newtonian Approximation of General Relativity (4 h)

§ 1 Field equations, geodesics and metric tensor in the post-Newtonian approximation (1.5 h)

Post-Newtonian approximation scheme: small parameters, validity and application fields. Field equations in the linear approximation relative to the non-Galilean components of the metric tensor. Estimates of the order of magnitude of the non-Galilean part of the metric. Non-isotropic geodesics in the weak field, slow motion approximation. Christoffel symbols of the second kind for the weak field approximation. Newtonian approximation of GRT: metric and equations of motion of a test particle. Metric tensor in the post-Newtonian approximation. Post-Newtonian gravitational potentials.

§ 2 Motion of a test particle in the post-Newtonian approximation (1 h)

Metric of one spherically symmetric static body. Post-Newtonian equations of motion of a test particle in the field of one spherically symmetric static body. Solution in osculating elements. Relativistic effects in the longitude of perihelion.

§ 3 Light propagation in the post-Newtonian approximation (1 h)

Isotropic geodesics in the weak field. Post-Newtonian equations of light propagation in the field of a spherically symmetric body. General scheme for solving the equations of light propagation. Initial value problem. Relativistic light deflection. Boundary value problem. Shapiro effect (time delay). Gravitational lensing.

§ 4 Treatment of observations in the post-Newtonian approximation (0.5 h)

Components of tetrad in the post-Newtonian approximation. Proper (observable) direction of light propagation and its relation to the coordinate velocity of light at the point of observation. Relativistic precession: geodesic, Lense-Thirring and Thomas precessions. Numerical estimates of the relativistic precessions.

IV Modeling of Astronomical Observations in General Relativity (4 h)

§ 1 General scheme of relativistic modeling of astronomical observations (1 h)

Parts constituting a typical astronomical phenomenon and the process of its observation. Modeling of the motions of the observer, the object of observations and electromagnetic signal between the object and the observer. The process of observation and its modeling. Confronting relativistic models with observations: parameters of the model and their possible dependence on the used reference system. The necessity to introduce several relativistic reference systems.

§ 2 Hierarchy of relativistic astronomical reference systems and time scales (1.5 h)

Global barycentric celestial reference system (BCRS): structure of the metric tensor, typical applications. Regional geocentric celestial reference system (GCRS): structure of the metric tensor and the coordinate transformations between BCRS and GCRS, basic properties of the GCRS and its typical applications. Local reference systems of an observer (Observer's RS – ORS), their properties and applications. Dynamically and kinematically nonrotating reference systems. Astronomical reference frames as a result of materialization of the corresponding relativistic astronomical reference systems. Ideal and real time scales. TCG and TCB as coordinate time scales of the BCRS and the GCRS. Interpretation of TCG and TCB as proper time scales of fictitious observers. The TCB–TCG transformation and its structure. Problem of the secular rate of time scales: TT (TDT) and TAI as practical realizations of TCG. Relation between proper time of an Earth-bound observer and TT. Possible scaling of the BCRS and GCRS spatial coordinates in accordance to TDB and TT, units of measurements. Difference in numerical values of masses when using TCG and TT. Synchronization of remote clocks in GRT: absolute simultaneity in Newtonian mechanics, relativity of simultaneity in SRT, Einsteinian synchronization, concept of coordinate synchronization in GRT. Sagnac effect. Example: generalization of the Einsteinian procedure for clock synchronization for GRT.

§ 3 Microarcsecond astrometry (1.5 h)

General scheme of relativistic reduction of positional observations. Modeling of local reference frame of the observer. Relativistic aberration. Comparison to Newtonian aberration. Magnitude of the relativistic aberration of the second and third orders. Gravitational light deflection with microarcsecond accuracy: monopole and multipole gravitational fields, rotational and translational motion. Parallax and proper motion in relativistic context.