LECTURE COURSE

Relativistic Foundations of Astrometry and Celestial Mechanics

1 semester: 28 hours

Introduction (2 h.)

Subject of relativistic astrometry and celestial mechanics. History of relativistic astrometry and celestial mechanics. Perihelion advance of Mercury. Suggested explanations of the excessive shift in the framework of Newtonian mechanics. Creation of Special Relativity Theory (SRT). Creation of General Relativity Theory (GRT). Classical tests of GRT. The development of GRS in the 20th–80th. Status of relativity theory in astronomy: GRT is the foundation of astrometry and celestial mechanics. What and how are we going to study. Recommended literature to the lectures.

I Essential Tensor Calculus and Riemannien Geometry (5 h.)

§ 1 Tensors in Euclidean space (2.7 h.)

Vectors in Euclidean space. Covariant and contravariant components of vectors. Metric tensor. General definition of tensor. Basic tensor operations. Differentiation of tensor fields: partial and covariant derivatives. Christoffel symbols of the first and second kinds. Absolute derivative. Parallel transport along a curve.

§ 2 Tensors in Riemannien space (1.5 h.)

Definition of a Riemannien space. Metric tensor of a Riemannien space. Tangent space. Geodetic lines. Curvature tensor, Ricci tensor, scalar curvature.

§ 3 Pseudo-Euclidean and pseudo-Riemannien spaces (0.8 h.)

The metric as quadratic form. Signature of metric. Euclidean and pseudo-Euclidean spaces. Riemannien and pseudo-Riemannien spaces. Pseudo-Euclidean and pseudo-Riemannien geometry.

II Foundations of General Relativity Theory (5 h.)

§ 1 Newtonian mechanics and Newtonian theory of gravitation (1 h.)

Space and time in Newtonian physics: uniformity of space and time, isotropy of space, Euclidean properties of space, absolute character of time. Inertial reference systems of Newtonian mechanics and Galilean principle of relativity. Relativity of space in Newtonian mechanics. Weak equivalence principle: proportionality of inertial and gravitational masses. Newtonian gravitational field equations. Infinite velocity of propagation of gravitational field in Newtonian physics.

§ 2 Basic ideas of Special Relativity Theory (2 h.)

Definition of inertial reference systems in SRT. Special principle of relativity. Interval and its invariance in all inertial reference systems, pseudo-Euclidean properties of event space in SRT. Minkowski metric. Lorentz transformation. Some consequences of the Lorentz transformations. World line of a particle. Isotropic (null), spacelike and timelike intervals and geodetics. Light cone. Absolute future, absolute past and absolute distant events. 4-velocity of a particle and its relationship to the 3-dimensional spatial velocity. Energy-momentum tensor in SRT. Conservation laws in SRT.

§ 3 Basic ideas of General Relativity Theory (2 h.)

Unique properties of gravitation as a force: Einstein's elevator. Einsteinian principle of equivalence. Relationship between Einsteinian and weak equivalence principles. Locally inertial reference systems. Pseudo-Riemannien properties of event space in GRT. Quasi-Cartesian coordinates. Field equations of GRT and how to "derive" them. Cosmological constant. Gauge conditions and their role to solve the GRT field equations. Harmonic reference systems. Equations of motion of test particles in GRT: geodetic principle and its relationship to the Einsteinian principle of equivalence. Light propagation in GRT: limit of geometrical optics.

III Post-Newtonian approximation of General Relativity (6.5 h.)

§ 1 Field equations, geodetics and metric tensor in the post-Newtonian approximation (2.5 h.)

Post-Newtonian approximation scheme: small parameters, validity and application fields. Field equation in the approximation linear relative to the non-Galilean components of the metric tensor. Estimates of the order of magnitude of the non-Galilean part of the metric. Non-isotropic geodetics in the weak field, slow motion approximation. Christoffel symbols of the second kind for the weak field approximation. Newtonian approximation of GRT: metric and equations of motion of a test particle. Metric tensor in the post-Newtonian approximation. Post-Newtonian gravitational potentials.

§ 2 Motion of a test particle in the post-Newtonian approximation (0.75 h.)

Metric of one spherically symmetric static body. Post-Newtonian equations of motion of a test particle in the field of one spherically symmetric static body. Solution in osculating elements. Relativistic effect in the longitude of perihelion.

§ 3 Light propagation in the post-Newtonian approximation (0.75 h.)

Isotropic geodetics in the weak field. Post-Newtonian equations of light propagation in the field of a spherically symmetric body. General scheme for solving the equations of light propagation. Initial value problem. Relativistic light deflection. Boundary value problem. Shapiro effect (time delay).

§ 4 Treatment of observations in the post-Newtonian approximation (2.5 h.)

Observable and non-observable (coordinate dependent) quantities. Modeling of observations in SRT. Test observers. Proper time. Locally inertial reference systems of an observer: tetrad formalism. Monad, triad and tetrad. Physical meaning of tetrad. Manipulations with tetrad indices. Equations defining the vectors of a tetrad. The use of tetrad for calculating interval of observable (proper) time and observable distance between two infinitesimally close events. Components of tetrad in the post-Newtonian approximation. Proper (observable) direction of light propagation and its relation to the coordinate velocity of light in the point of observation. Laws for the change of the vector of a tetrad for a moving observer. Tetrad as a coordinate basis of local reference system of an observer. Dynamically and kinematically nonrotating tetrads. Fermi-Walker transport. Relativistic precession: geodetic, Lense-Thirring and Thomas precessions. Numerical estimates of the relativistic precessions.

IV Exact solutions of the Einstein equations (1.5 h.)

§ 1 Schwarzschild solution (1.2 h.)

The problem of one attracting center in GRT. Schwarzschild metric: a sketch of the derivation of the metric, metric tensor in the standard Schwarzschild and harmonic coordinates. Physical equivalence of the metric tensors in various reference systems. Uniqueness of the Schwarzschild metric as a spherically symmetric solution of the Einstein field equations. Motion of a test particle in the Schwarzschild field: exact equation of the trajectory. Overview of the possible trajectories of test particles. Overview of the possible trajectories of the light. Horizon. Black holes.

§ 2 Kerr solution (0.3 h.)

Kerr metric in the Boyer-Lindquist coordinates. Uniqueness of the Kerr solution. Reissner-Nordtstrøm and Kerr-Newman solutions. Other exact solutions.

V Modeling of astronomical observations in General Relativity (8 h.)

§ 1 General scheme of relativistic modeling of astronomical observations (1 h.)

Parts constituting a typical astronomical event and the process of its observation. Modeling of the motion of observer, the object of observations and electromagnetic signal between the object and the observer. The process of observation and its modeling. Confronting relativistic models to observations: parameters of the model and their possible dependence on the reference system. The necessity to introduce several relativistic reference systems.

§ 2 Hierarchy of relativistic astronomical reference systems (1.5 h.)

Global barycentric reference system (BRS): structure of the metric tensor, typical applications. Regional geocentric reference system (GRS): structure of metric tensor and BRS–GRS transformations, basic properties of the GRS and its typical applications. Local reference systems (Observer's RS – ORS), their properties and applications. Dynamically and kinematically nonrotating reference systems. Astronomical reference frames as a result of materialization of the relativistic astronomical reference systems.

§ 3 Relativistic astronomical time scales (2.5 h.)

Ideal and real time scales. TCG and TCB as coordinate time scales of the BRS and the GRS. Interpretation of TCG and TCB as proper time of fictitious observers. The TCB–TCG transformation and its structure. Problem of rate of time scales: TT (TDT) and TAI as its practical realization. Relation between proper time of an Earth-bound observer and TT. TDB scale. Scaling of the BRS and GRS spatial coordinates in accordance to TDB and TT: units of measurements. Difference in numerical values of masses when using TCG and TT. Synchronization of remote clocks in GRT: absolute simultaneity in Newtonian mechanics; relativity of simultaneity in SRT; Einsteinian synchronization; concept of coordinate synchronization in GRT; example: generalization of the Einsteinian procedure for clock synchronization onto GRT.

§ 4 Relativistic perturbations in the motion of real celestial bodies (1 h.)

Coordinate dependence of the "relativistic perturbations". Relativistic perturbations in the motion of major planets and interplanetary vehicles. Relativistic perturbations in the motion of the Moon. Relativistic perturbations in the motion of artificial satellites of the Earth.

§ 5 Positional observations (1 h.)

General scheme of relativistic reduction of positional observations. Relativistic aberration. Comparison to Newtonian aberration. Magnitude of the relativistic aberration of the second and third orders.

§ 6 Range and Doppler observations (1 h.)

General scheme of relativistic reduction of range observations. Frequency from the point of view of GRT. General scheme of relativistic reduction of Doppler observations. Relativistic Doppler effect.

Recommended sources

Book written specially for astronomers:

1. V.A. Brumberg (1990) Essential Relativistic Celestial Mechanics. Adam Hilder, Bristol

   The most modern course. It contains a compact introduction into the mathematics needed for the GRT and in the physical foundations of the theory. Separate chapters are devoted to relativistic equations of motion of the Solar system, to the problems of relativistic reduction of astronomical observations as well as to relativistic effects in geodynamics (time scales, time synchronization, etc.)

2. V.A. Brumberg (1972) Relativistic Celestial Mechanics. Moscow, Nauka (in Russian)

   This is the previous version of the first book. Good introduction to the GRT itself and its mathematical language. The description of the astronomical applications of the GRT is out of date.

3. M. Soffel (1989) Relativity in Astrometry, Celestial Mechanics and Geodesy. Springer, Berlin

   The book is a little bit too complicated for beginners. The reader has to know the foundations of the GRT before reading the book. The books contains a lot of information about the relativistic effects in various astronomical observations.

Some textbooks for physicists:

1. L.D. Landau, E.M. Lifshitz (1971) The Classical Theory of Fields. Pergamon Press, Oxford
2. S. Weinberg (1972) Gravitation and Cosmology. John Wiley and Sons, New York
3. C.W. Misner, K.S. Thorne, J.A. Wheeler (1973) Gravitation. Freeman, San Francisco.
4. C.M. Will (1993) Theory and Experiment in Gravitational Physics. Cambridge University Press, Cambridge.