Celestial Mechanics

1 Semester: 28 Hours

I Introduction

Research field of celestial mechanics. Historical overview: apparent motion of planets, and solar and lunar eclipse as impetus for celestial mechanics. Ancient celestial mechanics. Appolonius and the idea of epicyclic motion. Ptolemy and the geocentric system. Copernicus and the heliocentric system. Kepler and the three Kepler laws. Galileo: satellites of Jupiter as a model for the Solar system, the begin of mechanics. Newton: mathematical formulation of mechanics, gravitational force. Einstein: the problem of perihelion advance of Mercury and the general theory of relativity.

Three aspects of celestial mechanics: physics of motion, mathematics of motion and (numerical) calculation of motion. The astronomical objects and specific goals and problems of the modelling of their motion: artificial satellites, the Moon, major planets, asteroids, comets, Kuiper belt objects, satellites of the major planets, rings, interplanetary dust, stars in binary and multiple systems, stars in star clusters and galaxies.

II Introduction into Maple

Brief history of computer algebra systems. Maple as an integrated computing environment for scientific calculations. The help system of Maple. Input and output. Assignments. Maple as a pocket calculator: numbers, operations, mathematical functions. Graphical capabilities: 2-dimensional plots, 3-dimensional plots, animation. Maple as a symbolic calculator: elementary algebraic manipulations, differentiation and integration, symbolic solution of equations, limits, sums, products, solutions of differential equations, substitutions. Maple as a library of numerical methods of computations. Maple as a computer language: control structures, functions, procedures.

III Two-body Problem

§ 1 Equations of motion

Equations of motion of one test body around a motionless massive body. Equations of general two-body problem. Center of mass. Relative motion of two bodies. Motion relative the center of mass.

§ 2 Integrals of angular momentum and energy

Angular momentum integral (the law of areas). The second Kepler's law. Integral of energy. Integrals of angular momentum and energy in polar coordinates.

§ 3 Orbit in Space

The three Euler angles: longitude of the ascending node, inclination and the argument of pericenter. The rotational matrix between inertial coordinates in space and the coordinates in the orbital plane.

§ 4 Kepler Equation

True Anomaly. Kepler equation in true anomaly. Eccentric anomaly. Various relations between the true and eccentric anomaly. Kepler equation in eccentric anomaly. Mean anomaly. The period of motion and the third Kepler law.

§ 5 Numerical Solution of the Kepler Equation

Existence and uniqueness of the solution. Iterative solution. Newtonian solution.

§ 6 Hyperbolic and Parabolic Motion

The eccentric anomaly and the Kepler equation for the hyperbolic motion. Explicit solution for the parabolic motion.

§ 7 Relation between Position, Velocity and the Kepler Elements

Calculation of position and velocity from the Kepler elements. Calculation of the Kepler elements from the position and velocity. Orbit determination (an overview).

§ 8 Series Expansions in Two-Body Problem

Series in powers of time. Fourier series in multiples of the mean anomaly. Series in powers of the eccentricity.

§ 9 Astronomical units of measurements

The SI units for mass, distance and time. The astronomical units of measurements. The relations between the SI and astronomical units.

IV The N-body problem

§ 1 Equations of motion

Equations of motion of the N-body problem. Gravitational potential.

§ 2 Classical Integrals of the N-body motion

Integrals of center of masses. Angular momentum integral in the N-body problem. Integral of energy in the N-body problem.

§ 3 The disturbing function

Planetary motion as perturbed two-body motion. The planetary disturbing function.

§ 4 Planetary ephemerides

Numerical ephemerides. JPL ephemerides. Analytical solutions: VSOP87, ELP82.

V Elements of the Perturbation Theory

§ 1 The method of the variation of constants

The variation of constants as a method to solve differential equations. Instantaneous elements. Osculating elements.

§ 2 Euler-Gauss equations

The radial, tangential and transverse components of the disturbing force. The Euler-Gauss equations: the differential equations for the osculating elements. Other variants of the Euler-Gauss equations.

§ 3 Lagrange equations

The potential disturbing force. The Lagrange equations for the osculating elements. Properties of the Lagrange equations.

VI Three-body problem

§ 1 The Lagrange solutions

The case when the three-body motion can be described by the equations of motion of the two-body problem: the five Lagrange solutions. Examples of the Lagrange motion in the Solar system.

§ 2 The restricted three-body problem

The equations of motion. Corotating coordinates. The equations of motion in the corotating coordinates. The Jacobi integral. The Hill's surfaces of zero velocity.

§ 3 Motion near the Lagrange equilibrium points

The Lagrange points as equilibrium points. Libration. The equations of motion for the motion near the Lagrange points. The stability of the motion near the equilibrium points

VII Gravitational Potential of an Extended Body

§ 1 Expansion of the Potential

Gravitational potential of an extended body as integral. The Laplace equation. Legendre polynomials. Associated Legendre polynomials. Expansion of the potential in terms of the associated Legendre polynomials.

§ 2 First terms of the expansion

Mass. Center of mass. The orientation of the coordinate systems.

§ 3 Symmetric bodies

Axial symmetry. Axial symmetry and the symmetry between the north and the south. Spherical symmetry.

§ 4 Spherical functions and the classification of the coefficients

Definition of spherical functions. The expansion of the gravitational potential in terms of the spherical functions. The principal part of the potential: spherically-symmetric gravitational field. Zonal coefficients. Sectorial and tesseral coefficients.

VIII Satellite Motion

§ 1 Typical perturbations in satellite motion

Gravitational field of the Earth. Gravitational forces of the Moon and the Sun. The planetary perturbations. Atmospheric drag. Light pressure. Magnetic field of the Earth. Neutral and charged particles. Relativistic perturbations.

§ 2 Motion in the quadrupole field

The disturbing function for the oblate Earth. Solution in osculating elements. Secular perturbations. Numerical example. Periodical perturbations.

§ 3 Atmospheric drag

The model of the perturbing force. Models for the atmospheric pressure. The Euler-Gauss equations for the atmospheric drag. Averaging. The simplified Euler-Gauss equations for small eccentricities. Solution and its properties.

IX Numerical integration of ordinary differential equations

§ 1 Basic notions

Euler step for the differential equations of the first order. Discretization. Three kinds of errors: the local truncation error, the global error and the roundoff error.

§ 2 Methods of numerical integration

The method of Taylor expansion. The Runge-Kutta method. Stepsize control for the Runge-Kutta methods (Fehlberg method). The Runge-Kutta-Nyström method. Multistep methods. Explicit and implicit methods. Predictor-corrector methods. Adams-Bashforth and Adams-Moulton methods. Extrapolation methods.

§ 3 Reliability of numerical integration

Close encounters. Regularization. Accuracy control.

Literature

- A.E. Roy, Orbital Motion, 1988, Institute of Physics Publishing, Bristol and Philadelphia
- M. Schneider, Himmelsmechanik, 1984, B.I.-Wissenschaftsverlag, Mannheim
- O. Montenbruck, E. Gill, Satellite Orbits, 2000, Springer, Berlin
- A. Guthmann, Einführung in die Himmelsmechanik und Ephemeridenrechnung, 2000, Spektrum, Heidelberg